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Banco de México

Working Papers

N° 2017-04

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April 2017

Documento de Investigación
2017-04

Working Paper
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Commodity Price Risk Management and Fiscal Policy in a Sovereign Default Model

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Abstract: Commodity prices are an important driver of fiscal policy and the business cycle in many developing economies. We analyze a dynamic stochastic small-open-economy model of sovereign default, featuring endogenous fiscal policy and stochastic commodity revenues. The model accounts for a positive correlation of commodity revenues with government expenditures and a negative correlation with tax rates. We quantitatively document the extent to which the utilization of different financial hedging instruments by the government contributes to lowering the volatility of different macroeconomic variables and their correlation with commodity revenues. An event analysis illustrates how financial hedging instruments moderate fiscal adjustment in response to significant falls in the price of commodities.
Keywords: commodity revenues, hedging, indexed bonds, fiscal policy, sovereign default
JEL Classification: F34, F41, F44

Resumen: Los precios de las materias primas son un determinante importante de la política fiscal y del ciclo económico en numerosas economías en desarrollo. Analizamos un modelo dinámico y estocástico de una economía pequeña y abierta con incumplimiento de pago de deuda soberana, considerando política fiscal endógena e ingresos estocásticos de materias primas. El modelo explica una correlación positiva de los ingresos por materias primas con el gasto del gobierno y una correlación negativa con la tasa de impuestos. Documentamos de manera cuantitativa en qué medida la utilización de diferentes instrumentos financieros de cobertura por parte del gobierno contribuye a reducir la volatilidad de diferentes variables macroeconómicas y su correlación con los ingresos por materias primas. El análisis de eventos ilustra cómo los instrumentos financieros de cobertura moderan los ajustes fiscales en respuesta a caídas significativas en el precio de las materias primas.
Palabras Clave: ingresos por materias primas, coberturas, bonos indexados, política fiscal, incumplimiento de pago de deuda soberana

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1 Introduction

Commodity prices are an important driver of the behavior of fiscal policy and the business cycle in commodity exporting developing and emerging market economies.[1] Among other factors, these results have been attributed to the fact that governments in many economies rely to an important extent on commodity revenues to finance their budgets. For example, in more than twenty countries hydrocarbon revenues account for over thirty percent of total fiscal revenue (IMF, 2007). Given their reliance on a highly volatile source of revenue, these economies face a significant challenge in terms of their capacity to smooth fiscal policy and fluctuations in economic activity.

Different instruments have been proposed and implemented with the purpose of moderating the impact of commodity-price fluctuations on public finances. In this article we exploit a dynamic model of sovereign default with endogenous fiscal policy, introducing a stochastic endowment of commodity-revenues for the government, to contribute to our understanding of the potential macroeconomic consequences of using these instruments.[2] This model is a natural framework to illustrate the trade-offs faced by a government subject to significant fluctuations in commodity-related revenues as it endogeneizes the decisions of public expenditures, distortionary tax rates, the issuance of debt and the default of sovereign debt. Furthermore, it allows us to do so in a relatively standard business cycle environment.

In our framework, fluctuations in commodity prices affect the economy through their impact on the government budget constraint and the ability of the government to access credit in international financial markets. As is standard in the sovereign default literature (which we discuss below), the incentives for default increase when income is low, which occurs when aggregate productivity or when the price of commodities is low, or a combination of both. In these situations the likelihood of default increases and investors

[1]For empirical evidence see Medina (2010), Villafuerte and Lopez-Murphy (2010), Spatafora and Samake (2012), Cespedes and Velasco (2014), Talvi and Vegh (2005), Fernandez et al. (2015). Husain et al. (2008) and Pieschacon (2012), among others, assert that fiscal policy is the key mechanism through which oil prices affect the economic cycle in oil-exporting countries. For discussions and evidence of procyclical fiscal policy in developing economies see Ilzetzki and Vegh (2008), Talvi and Vegh (2005), Frankel et al. (2013) and references therein. Vegh and Vuletin (2012) find that tax rate policy is procyclical in most developing countries (tax rates are negatively correlated with GDP), but acyclical in industrial countries.

[2]Mendoza (1995) finds that terms-of-trade shocks account for approximately 1/2 of GDP variability in a dynamic stochastic small open economy framework. Shousha (2016) estimates that innovations in real commodity export prices are responsible for 23 percent of movements in aggregate output in emerging economies. Schmitt-Grohé and Uribe (2015) estimate that terms-of-trade shocks explain approximately 10 percent of movements in aggregate activity in less developed and emerging economies while Fernandez et al. (2015) estimate a model that assigns to commodity shocks 42 percent of the variance in income.

are less willing to increase lending to the government, which induces the government to rely more heavily on taxation to provide for expenditures. Since taxes are distortionary, higher taxes induce lower labor input and thus lower non-resource output and private consumption. By calibrating the model to Mexico, we are able to assess the quantitative performance of this mechanism.[3]

We use our framework to evaluate the business cycle and welfare implications of the utilization by the government of different financial instruments that serve the purpose of moderating fluctuations in commodity-revenues: commodity indexed bonds and financial derivatives.[4] We quantify how these instruments generate a reduction in the volatility of different macroeconomic variables and their correlation with commodity prices. We then compare how the economy reacts, under alternative scenarios, to significant drops in the price of commodities and illustrate how hedging instruments allow for a relatively smooth adjustment of fiscal variables.

The use of this type of financial instruments is, of course, not a recent idea in the academic literature or in economic policy. In 1782, the State of Virginia issued bonds linked to the price of land and slaves. In 1863 the Confederate States of America issued bonds payable in pounds sterling or French francs but convertible into cotton at a predetermined price (Borensztein and Mauro, 2004). More recently, both sovereign countries and corporate entities have issued debt linked to the price of different commodities including gold and silver as well as oil (for a summary of these experiences see Atta-Mensah, 2004). The government of Mexico is believed to be the first to issue oil-linked bonds during the 1970s, known in financial markets as Petrobonds.[5] Nigeria and Venezuela issued oil-linked bonds in the 1990s in exchange for defaulted loans (Sandleris and Wright, 2013). The World Bank has made available loans combined with protection from commodity price fluctuations, although with limited use (Borensztein and Mauro, 2004). There is also experience with the use of financial derivatives. The federal government of Mexico

[3]Although we focus on the case of oil-revenues for Mexico, the implications derived from our model can be generalized to economies that rely on different commodities for a significant proportion of their fiscal revenues. As we examine below, the model is able to replicate well the relationships between oil prices and government expenditures reported by Pieschacon (2012) using a VAR approach for Mexico, the cyclical behavior of tax rates as documented by Vegh and Vuletin (2012) for emerging economies and other regularities supported by the empirical literature.

[4]We analyze the following scenarios: (1) baseline model with standard non-contingent bonds, (2) non-contingent bonds and use of *forward sale* of commodities, (3) non-contingent bonds and use of sale options for commodities, (4) commodity indexed bonds. We view these instruments as complementary to the role of stabilization funds aimed at smoothing fluctuations in international commodity prices (for a discussion see Daniel, 2001). In principle, hedging strategies could reduce the need to accumulate wealth in stabilization funds and the cost of opportunity they imply.

[5]http://www.banxico.org.mx/divulgacion/sistema-financiero/sistema-financiero.html

has used financial risk management tools to hedge the risk of fluctuations in the price of oil at least since the early 1990s (Daniel, 2001; IMF, 2007), and in a systematic manner since 2004. This experience suggests that producers can successfully exploit hedging opportunities provided by financial markets.

Most previous work in the literature has focused on studying the potential gains of issuing GDP-indexed sovereign debt. This consists of financial instruments that specify payments according to the outcome of GDP. Therefore, a government that issues these bonds faces lower debt payments during economic downturns, which can potentially facilitate countercyclical fiscal policy and diminish the likelihood of fiscal crises as well as contribute to reduce the volatility of macroeconomic variables.[6] Hatchondo and Martinez (2012), for example, introduce output-indexed bonds into an equilibrium sovereign default model and calculate that the welfare gain from the introduction of these instruments is equivalent to 1/2 a percentage point of consumption.[7] The gains come from the result that these bonds allow the government to avoid costly default episodes, increase the levels of debt and improve consumption smoothing.

The use of commodity-indexed debt instruments and financial derivatives has been proposed by authors such as Daniel (2001) and Atta-Mensah (2004), among many others (see references in Borensztein and Mauro, 2004). Malone (2005) evaluates the benefits of using financial derivatives to hedge commodity price risk in a stylized two-period default model economy. Caballero and Panageas (2008) work with a sudden-stop model to study the case of Chile, where the business cycle is influenced by the price of copper, and argue that existing financial markets could be exploited to hedge against variations in the likelihood of a sudden-stop. Borensztein et al. (2015) analyze the welfare gains that a small open economy can derive from insuring against natural disasters with *catastrophe bonds*.

Perhaps closest to our work is that of Borensztein, Jeanne and Sandri (2013). They also analyze the potential welfare gains of hedging against commodity price risk for commodity exporting economies. In their model, hedging enhances welfare through two channels: first, by reducing export income volatility; and second, by reducing the need to hold

[6]This literature includes Borensztein and Mauro (2004), Sandleris et al. (2008).

[7]This estimate could be considered an upper bound on the potential gains of GDP-indexed debt instruments since, among other assumptions, they specify a portfolio of complete Arrow-Debreu securities instead of a single output-indexed bond. Their formulation eliminates sovereign default and its associated costs in equilibrium (which does occur without indexed bonds) given that foreign investors will not purchase assets that are contingent on a realization of GDP that results in default. As noted in the literature, there may be obstacles to issuing this type of debt instruments, such as the possibility that GDP may no be easily verifiable. GDP-indexed bonds have also been considered within a debt sustainability framework (for a discussion see Hatchondo and Martinez, 2012).

precautionary reserves and improving the ability of the country to borrow against future export income. The contribution in our article is to evaluate the use of financial instruments in a small open economy general equilibrium framework where tax rates, government expenditures and debt levels are endogenous and depend on stochastic aggregate productivity as well as oil-revenues. In addition to being able to study the behavior of different fiscal variables in our model, we further the analysis in Borensztein et al. (2013) by considering an economy with endogenous non-resource output subject to productivity shocks, as opposed to an exogenous fixed level. This opens the possibility of assessing a transmission mechanism from fiscal policy to the private sector.[8] Additionally, in our model the possibility of sovereign debt default determines endogenously how much investors are willing to lend to the government, and this potentially depends on the capacity of the government to moderate the volatility of commodity-related revenues. We show that, in contrast to Borensztein et al. (2013), in our model there is not significant room for increasing average debt levels, as this is mainly determined by the cost-benefit trade-offs implied by default. In our model the costs of default are generated by a loss in output and temporary exclusion from financial markets while the volatility of commodity revenues plays a relatively limited role in the determination of debt levels.

Our exposition proceeds as follows: the economic environment and the description of our theoretical framework are provided in Section 2, we discuss the main mechanisms underlying our model in Section 3. The parameterization and calibration approach are described in Section 4. In Section 5 we outline the different financial instruments that are introduced into the model. Section 6 presents the quantitative analysis and our main results. Section 7 provides a final discussion and the conclusion.

2 Quantitative Framework

We exploit the canonical model of sovereign default of Cuadra et al. (2010), in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). Relative to alternative models in the sovereign default literature, the model features endogenous government expenditures (separated from private consumption), tax rates and debt levels as well as endogenous household labor supply. Considering an elastic labor supply allows tax rates to have a distortive effect on non-resource production, which represents the transmission mechanism from government policy to the private sector in our model. We introduce an exogenous stochastic endowment of commodity revenues for the government.

[8] We discuss an alternative potential transmission mechanism in the conclusion.

The environment consists of a small open economy model with three agents: a representative household, the government and international lenders. The representative household values private consumption, government spending and leisure. In every period the household makes a decision on labor supply taking as given the tax rate set by the government and the aggregate productivity shock. The government maximizes the welfare of the household and has access to international financial markets where it can borrow by issuing a one-period non-contingent bond. The government also decides on the level of public spending and borrowing as well as the level of the tax rate. Furthermore, it can decide to default on its debt obligations, which results in a loss in output and temporary exclusion from credit markets. Lenders charge a premium on the interest rate paid by the government, which is based on the expected probability of default and a stochastic discount factor. The stochastic discount factor is motivated by the observation that if governments tend to default when investors have high marginal utility then bond prices reflect compensation for this risk.[9]

2.1 Households and Production Technology

There is a representative household with present expected discounted value of future utility flows represented by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\, u(c_t, g_t, 1-l_t)\right] \tag{1}$$

where the discount factor is given by β and the per period utility function is specified in the following manner:

$$u(c_t, g_t, 1-l_t) = \pi\,\frac{g_t^{1-\sigma}}{1-\sigma} + (1-\pi)\,\frac{(c_t - l_t^{1+\psi}/(1+\psi))^{1-\sigma}}{1-\sigma} \tag{2}$$

The representative household values private consumption c_t, public expenditures g_t and leisure $1 - l_t$. Utility is separable in private sector variables c_t and l_t and public expenditures g_t. Parameter π determines the weight given to government expenditures in the utility function. Parameter ψ governs the elasticity of the supply of labor by the household with respect to the return to labor; in our model this return will be determined by the exogenous aggregate productivity level and the tax rate set by the government (under the

[9]There is evidence that the risk premium is an important factor in accounting for the behavior of sovereign bond prices (see Lizarazo, 2013).

utility function specification the marginal rate of substitution between private consumption and labor is independent of consumption). The coefficient or relative risk aversion is set by parameter σ.

There is a tradable good produced using labor services with a production technology that is subject to productivity shocks $y_t = a_t\, f(l_t)$, where productivity a takes on a finite number of values defined over the set S_a and evolves according to a transition matrix denoted by $\Lambda(a' \mid a)$. Private consumption is taxed by the government, the representative household makes private consumption and leisure decisions based on the budget constraint $(1+\tau)\, c_t = a_t\, f(l_t)$, where τ is the tax rate set by the government at the beginning of the period.[10] The optimal household decisions are written as $c^*(a,\tau)$ and $l^*(a,\tau)$.

2.2 The Dynamic Problem of the Government

The government maximizes the welfare of the representative household. In every period the government makes decisions regarding the levels of debt it issues in international financial markets, the tax rate and government expenditures. Additionally, the government can decide to default on its debt.

When the government has access to financial markets, the dynamic problem can be written in recursive form as follows:

$$v_c(b,a,z) = \max_{\{g,b',\tau\}} u(c^*, g, 1-l^*) + \beta \sum_{\{a',z'\}} \Lambda(a' \mid a)\, \Gamma(z' \mid z)\, \overline{v}(b',a',z') \tag{3}$$

subject to the optimal household functions $c^*(\tau,a)$ and $l^*(\tau,a)$, which the government takes as given. The government budget constraint is $g = \tau\, c + b - q(b',a,z)\, b' + x$, where b is the level of foreign assets (primes denote variables for the next period) and x are commodity-revenues denominated in units of the tradable good. The price of bonds is a function $q(b',a,z)$ that is endogenously determined in equilibrium and discussed below. Commodity-revenues $x = \theta \cdot z$ take on a finite number of values defined over the set S_x, where θ is a (fixed) parameter that determines the average level of revenues and z is the

[10]The government is the only domestic agent in this economy that can hold assets or borrow. In developing economies government debt can account for most of external debt. In the case of Mexico, for example, during the financial crisis of 1995, sovereign external debt accounted for almost 70 percent of the total stock of foreign debt (see Cuadra et al. 2010). Furthermore, the volume of total credit to the private sector has been historically low in Mexico (see Lopez-Martin, 2016), as is typical in many developing economies. Additionally, we use the volatility of consumption as a target in our calibration procedure. We abstract from the direct exposure of the private sector to commodity prices.

exogenous stochastic price of the commodity. This price is defined over the set S_z and evolves according to a discrete transition matrix process denoted by $\Gamma(z' \mid z)$.[11] The possibility of default is introduced in the expression $\overline{v}(b,a,z) = \max\{v_c(b,a,z), v_d(a,z)\}$, where $v_d(a,z)$ is the value of default and $v_c(b,a,z)$ is the value of maintaining access to international credit markets.

When the government decides to default it temporarily loses access to international credit markets and its budget constraint becomes $g = \tau c + x$. Additionally, there is an efficiency loss in aggregate productivity represented by the function $h(a) \leq a$ (the rationale for this loss and the calibration approach are described in the calibration section). With no access to financial markets the dynamic problem of the government is given by expression (4):

$$v_d(a,z) = \max_{\{g,\tau\}} u(c_d^*, g, 1-l_d^*) + \beta \sum_{\{a',z'\}} \Lambda(\,\cdot\,)\,\Gamma(\,\cdot\,)\left\{\mu\,\overline{v}(b'=0,a',z') + (1-\mu)\,v_d(a',z')\right\}$$

subject to the budget constraint under default previously described and the optimal decisions of the household $c_d^*(\tau,a)$ and $l_d^*(\tau,a)$, when there is no access to international credit markets. The government regains access to financial markets with probability μ.[12]

2.3 International Lenders and the Price of Sovereign Bonds

The price of sovereign bonds is determined according to a no-arbitrage condition that incorporates a stochastic discount factor. International lenders have perfect information of the state of the economy (productivity, oil price and level of foreign assets) and take into account the endogenous probability that the government will default in the

[11] We abstract from fluctuations in quantities and focus on the risk implied by price volatility, for many commodities most of revenue fluctuations are accounted for by price variations (see Bems and de Carvalho Filho, 2011; Borensztein et al., 2013). Spatafora and Samake (2012) carry out a variance decomposition of commodity export revenues, their results suggest that prices largely drive changes in commodity export revenues: considering fuel-exporting developing countries during the period 1990-2010 the pure price effect accounted for 73.5 percent of the variance, while the pure volume effect and the correlation component accounted for 10.4 and 16.2, respectively (their Table 13). For the period 2000-2010 the pure price effect accounted for 77.5 of the variance of commodity export revenues. Pieschacon (2012), employing a VAR methodology for Mexico, finds no significant relationship between oil prices and oil production (her Figure 2).

[12] To keep our model tractable (which extends a standard sovereign default model along several dimensions), we follow most of the sovereign default literature in making the assumption that after default the economy eventually returns to financial markets with no debt burden (Arellano, 2008; Aguiar and Gopinath, 2006; Hatchondo et al., 2010; Mendoza and Yue, 2012). Yue (2010) introduces post-default debt renegotiation in an endowment sovereign default model that endogenizes debt recovery rates. It is left for future research to incorporate this debt renegotiation channel in a model with commodity revenues to asses the contribution of commodity prices to sovereign interest rate volatility.

next period to determine the price of the bond. The government takes as given the bond price function when selecting the level of debt. The consideration of a stochastic discount factor is motivated by the observation that spreads in developing and emerging market economies are higher during times of high risk aversion for international investors.

The specification we employ follows closely that of Arellano and Ramanarayanan (2012). The stochastic discount factor is given by $M(a_{t+1}, a_t) = \exp(-\vartheta_t\, \varepsilon_{t+1} - \frac{1}{2}\, \vartheta_t^2\, \sigma_a^2)$, where ε_{t+1} is the shock to aggregate productivity and σ_a^2 is its variance. The term $\vartheta_t = \alpha + \delta \log a_t$ depends on the state of aggregate productivity, allowing for time variation in the market price of risk. The risk premium is generated from the interaction of the stochastic discount factor and the expected probability of default. More explicitly, the price of the bond is determined by the following equation:

$$q(b', a, z) = \sum_{\{a', z'\}} M(a', a)\, \Lambda(a' \,|\, a)\, \Gamma(z' \,|\, z)\, (1 - d(b', a', z'))/(1 + r_f) \qquad (5)$$

where $d(b', a', z')$ is a function that equals one in the states where the government defaults and zero otherwise, r_f is the international risk free rate at which international lenders can borrow or lend.[13] Expression (5) can be rewritten as $q = \mathbb{E}\,[M' \cdot (1 - d')] \cdot (1 + r_f)^{-1}$, alternatively:

$$q = E\,[M'] \cdot E[1 - d'] \cdot (1 + r_f)^{-1} + Cov(M', 1 - d') \cdot (1 + r_f)^{-1}$$

which implies that if payoffs exhibit negative correlation with the pricing kernel, then a lower bond price q is required to compensate investors for this risk. With positive α, then ϑ_t is positive on average, which generates negative correlation between M and payoffs; negative shocks to productivity (which imply lower income) reduce the probability of repayment and future prices while increasing M, and thus bond prices have to be lower to compensate the risk for the investor. Additionally, with $\delta < 1$ the risk premium has to be higher when the borrower has low income.[14]

[13] The specification we employ slightly differs from Arellano and Ramanarayan (2012) since output is an exogenous process in their model (for a related specification see Hatchondo et al., 2012).

[14] As discussed in Arellano and Ramanarayanan (2012), this is a parsimonious specification for the modelling of risk premia, which does not require the introduction of additional state variables. In line with empirical evidence, it captures the behavior of higher sovereign debt spreads in times of high risk aversion for international investors. Lizarazo (2013) introduces risk averse investors who trade with the emerging economy; interest rates and capital flows are a function of fundamentals of the economy but also a function of financial wealth and risk aversion of international investors. We will consider the stochastic discount factor in the valuation of sale options.

2.4 Definition of Equilibrium

A **recursive equilibrium** of this small open economy is given by: value functions $v_c(b, a, z)$, $v_d(a, z)$ and $\overline{v}(b, a, z)$, the household's policy functions for consumption and labor: $c^*(\cdot)$, $c^*_d(\cdot)$, $l^*(\cdot)$ and $l^*_d(\cdot)$, the government's policy functions for asset/debt holdings $b'(b, a, z)$, its default decision $d(\cdot)$, government expenditure policy functions $g_c(\cdot)$ and $g_d(\cdot)$ (with access to international credit markets and under default, respectively), tax rate functions $\tau_c(\cdot)$ and $\tau_d(\cdot)$ and a bond price function $q(\cdot)$, **such that**: (i) given the government's policy functions and the bond price function, the household's policy functions solve its static optimization problem, (ii) given the bond price function and the household's policy functions, the government's policy functions and the value functions solve its dynamic problem, (iii) the bond price function $q(\cdot)$ is determined by the corresponding pricing equation.

3 Model Mechanics

In this section we briefly discuss the intuition behind the main mechanisms of the model.[15] Fig. 1 shows the default areas as a function of debt levels, aggregate productivity and commodity prices. As is standard in these models, default is more likely with more debt; this result follows from the property that the value of remaining in credit markets is decreasing with debt, while the value of default is independent of debt. Additionally, default incentives are stronger with lower aggregate productivity levels and lower commodity prices. Because of increasing and concave utility functions, net repayment is more costly when consumption is low, which results in default being relatively more attractive.

Fig. 2 shows the optimal tax rates as a function of debt levels, aggregate productivity and oil price shocks. When the price of oil is favorable the optimal tax rate is negatively correlated to the level of productivity: in states with low aggregate productivity access to international credit markets is relatively limited (as the likelihood of default increases), and the optimal level of the tax rate increases in order to finance government expenditures (left panel, Fig. 2). When the price of oil is less favorable, the level of government indebtedness is key in determining the relation between the optimal tax rate with aggregate productivity shocks. With a low productivity shock and at a low level of debt, the government can resort to borrowing and reduce the distortive effect of taxation at an already negative situation for private production and consumption due to low

[15]The discussion in part follows Arellano (2008) and Cuadra et al. (2010). Figures in this section are constructed with the calibrated model parameters enumerated below.

productivity. As indebtedness increases, the possibility of default becomes more likely, becoming more difficult for the government to access borrowing in a situation of low aggregate productivity, and therefore the government has to depend more on taxation to finance expenditures.[16]

The model generates a procyclical behavior of government expenditures as a result of the weak insurance role provided by the incomplete asset market structure which results in the relative difficulty of borrowing in lower income states. The transmission channel from government policy to private sector variables is the tax rate.[17] Tax income is necessary to finance government expenditures, but distorts the supply of labor and reduces private consumption.[18] Given that tax rates are higher with lower oil prices (see Figure 2), oil prices will be positively correlated with non-primary production.

4 Parameters and Functional Specifications

In this section we discuss the predetermined parameters for the model as well as our calibration approach for Mexico.

4.1 Predetermined Parameters

For setting the value of several parameters we take guidance from the literature (see Table 1). A standard value for the risk aversion parameter σ is 2. The discount parameter β is typically set between 0.95 and 0.97 for yearly specifications in business cycle models for developing countries (Pallage and Robe, 2003), but can be well below 0.85 in annual terms if taken from sovereign default models calibrated at a quarterly frequency (see Aguiar and Gopinath, 2006; Hatchondo et al., 2010; Yue, 2010).[19] These models typically require relative impatience to generate default in equilibrium (Hatchondo et al., 2009; Hatchondo et al., 2012), which has been associated to political factors, among others, in developing and emerging market economies. Our baseline parameterization is based on the latter approach, but we conduct robustness exercises and find that our results

[16]This mechanism is similar for a model with no oil revenues; see Cuadra et al. (2010), their Figure 4.

[17]A potential alternative financial transmission mechanism is through interest rates (for example see Tavares 2015). We leave this alternative channel for further research.

[18]More specifically, given the specified utility function, the labor supply of the household is given by $l = (a/(1+\tau))^{1/\psi}$.

[19]Reports from the Auditoría Superior de la Federación (ASF) describe the yearly nature of the hedging strategy in the case of Mexico. We calibrate our model in annual terms.

are qualitatively similar (we also consider a higher discount factor of 0.95 in the welfare analysis below).

Table 1. Predetermined Baseline Parameters.

description of parameter	parameter	value
risk aversion	σ	2.00
discount factor	β	0.85
labor elasticity	ψ	1/2.2
risk free interest rate	r_f	0.02
financial markets re-entry probability	μ	1/3
loss of aggregate productivity in default	ϕ	0.99
stochastic discount factor parameter	δ	-141
stochastic discount factor parameter	α	11
autocorrelation oil price	ρ_z	0.940
volatility oil price shocks	σ_z	0.230
autocorrelation aggregate productivity	ρ_a	0.900

Parameter ψ that determines labor elasticity is set equal to 0.455, while μ equal to 1/3 implies than on average countries in default return to international financial markets after 3 years (Cuadra et al., 2010). In the baseline calibration the risk free interest rate r_f is 0.02 in annual terms, an intermediate compromise value that takes into account its levels in recent years. The values for the stochastic discount factor parameters α and δ are taken from Arellano and Ramanarayanan (2012). There is a broad range of values in the literature for the autocorrelation parameter of the aggregate productivity process $\Lambda(a' \mid a)$, we initially set ρ_a equal to 0.90, in line with Arellano and Ramanarayanan (2012).

The persistence and volatility parameters, ρ_z and σ_z respectively, for the stochastic process of oil prices are from Borensztein et al. (2013), but we modify the AR(1) process as discussed below.[20] As discussed by Pieschacon (2012), it is not relevant for our purposes whether oil prices are driven by international supply or demand, as long as they are not significantly influenced by the behavior of the small open economy.

The aggregate productivity cost of default consists of a function $h(a)$ such that $h(a) = a - \omega$ when $a \leq \phi\,\bar{a}$, where ϕ is a parameter and $\bar{a}$ is the unconditional mean

[20]There is no consensus in the literature with regards to the stationarity of oil prices. The stationarity of real oil prices has been supported in the macroeconomics literature, and we follow this approach (for thorough discussions and related specifications see Bems and de Carvalho Filho, 2011; Pieschacon, 2012; Borensztein et al., 2013).

of productivity. When $a \geq \phi\,\bar{a}$, then $h(a) = \phi\,\bar{a} - \omega$. Relative to Arellano (2008), we introduce a parameter ω, to match the ratio of debt to output in Mexico (see Table 2).[21] This parameter shifts the level of productivity during default while maintaining the shape of the original function $h(a)$. We take the value of parameter ϕ from Cuadra et al. (2010) and set ω to match the ratio of net debt of the public sector to output during the 20 year period 1995-2014. The assumption that a default event is associated with output loss is standard in the literature and intends to capture, in a tractable manner, disruptions in economic activity. When default is more costly, according to the efficiency loss specification, higher levels of debt are sustained in equilibrium.[22]

4.2 Calibration

The model is parsimonious in terms of the number of parameters we need to calibrate (Table 2). We set the value for π to target the average tax rate, which is endogenous in our model. This parameter is the weight given to government expenditures in the utility function and it governs the extent to which the government is willing to distort the economy through taxation in order to provide this type of consumption. We make use of the estimates of the average effective tax rates on consumption and labor income by Anton-Sarabia (2005) for Mexico: consumption tax rates are roughly between 7 and 14 percent, while labor income tax rates are between 8 and 12.5 percent. As our target we take the lower bound of the total wedge implied by these estimates to keep the ratio of total government expenditures to output in line with the data.

[21] It is well known that sovereign default models face difficulty in jointly matching several moments related to sovereign debt and interest rates: average and volatility of sovereign interest rate spreads, frequency of default and average debt levels (see Aguiar and Gopinath, 2006; Arellano, 2008; Hatchondo and Martinez, 2009; Hatchondo et al., 2010; Yue, 2010; Roldan-Peña, 2012; Arellano and Ramanarayanan, 2012; Lizarazo, 2013). For example, Arellano (2008), obtains an average ratio of debt to output of 6 percent, while Yue (2010), who introduces debt renegotiation after default obtains a debt ratio of 10 percent. We opt, in our baseline calibration, to target the average debt level given our primary interest in evaluating how the risk generated by commodity prices affects debt levels and fiscal policy. In the Appendix we explore an alternative specification for efficiency losses during default.

[22] Mendoza and Yue (2012) propose a mechanism that generates an endogenous efficiency loss during default episodes: some imported inputs require working capital financing and in a default episode these inputs are replaced by imperfect substitutes as both government and firms are excluded from credit markets. Alonso-Ortiz et al. (2015) discuss how part of the sovereign-default literature coincides in setting the cost of default at a fall in aggregate productivity of around 5 percent. They use a calibrated continuous time sovereign default model where government default may trigger a change in the regime of a stochastic productivity process and find evidence in favor of productivity falls in the range of 3.7-5.9 percent. Furceri and Zdzienicka (2012) use an unbalanced panel of 154 countries from 1970-2008 and estimate that debt crises reduce contemporaneous output growth by about 6 percentage points (with different datasets and methodologies the magnitude of the effect ranges from 5 to 10 percentage points).

Parameter θ is set to match the average ratio of government oil-related revenues to output during the period 2004-2014; a value of 0.074 generates an average ratio of oil-revenues to total output of 0.081. Parameter σ_a drives aggregate volatility in this economy, our chosen target is the volatility of consumption (logged and detrended with the Hodrick-Prescott filter, as computed in Mendoza, 2010).

Table 2. Baseline Calibration.

description of parameter	parameter	value
utility weight on govt. expenditures	π	0.500
loss of aggregate productivity in default	ω	0.054
average level of govt. oil revenues	θ	0.074
probability large oil drops	λ	0.350
volatility aggregate productivity shocks	σ_a	0.005
target statistics	data	model
average total tax wedge	0.155	0.156
average level govt. oil revenues/output	0.081	0.081
volatility of consumption	3.397	3.870
average debt/output ratio	0.252	0.247
frequency large oil drops (per decade)	see text	see text

We first construct the Markov matrix $\Gamma(z' \mid z)$ as a discrete approximation of an AR(1) process for oil prices following Tauchen (1986). We then modify this matrix, with 5 grid-points, by adding probability to drops in oil prices: for the two highest levels in z we subtract λ from the probability that z remains unchanged in the following period and add this probability to the grid-point below each of these two values of z.[23] We do this to increase the frequency of large drops in oil prices and bring it closer to the data. IMF (2015) documents episodes where the rolling 12-month fall in oil prices exceeded 30 percent (approximately a one standard deviation event): two episodes during the 1980s, two episodes during the 1990s, two episodes during the 2000s, and one in 2015 (see their Figure 2). With λ equal to 0.35, our model simulations generate oil price falls larger than 30 percent at an average frequency of 1.2 times per decade (we consider falls when the price of oil is at or above the unconditional mean, which we examine in our event analysis), this is conservative in terms of the frequency of large drops in the price of oil.

[23]For example, $\Gamma(z_5 \mid z_5)$ is the transition probability that the price of oil remains at its highest level z_5, according to the method by Tauchen (1986) for discretizing an AR(1) process. Then we subtract λ in the final (modified) transition matrix and add this value to $\Gamma(z_4 \mid z_5)$.

5 Alternative Financial Instruments

We describe the introduction and specification of alternative financial instruments made available to the government.

5.1 Commodity-Indexed Bonds

We allow the government to issue bonds that promise to pay (in the case of no default) in the next period a coupon $\nu \cdot 1$ (which represents the fixed payment) plus $(1-\nu) \cdot z$ (the variable payment linked to the price of oil z). Parameter $\nu \in [0, 1]$ determines the extent to which debt is indexed to commodity prices: in the baseline specification for bonds ν equals one. A proportion $(1-\nu)$ of the payment promised by the bond is indexed to the price of the commodity z. The price of the commodity indexed bond is now given by:

$$q_z(b', a, z) = \sum_{\{a', z'\}} M(a', a)\, \Lambda(a' \mid a)\, \Gamma(z' \mid z)\, \{n(z) \cdot (1 - d_z(b', a', z'))/(1 + r_f)\}$$

where $n(z) = \nu + (1-\nu)\, z$ and $d_z(\,\cdot\,)$ is the default decision when the government issues commodity indexed debt. The budget constraint of the government is now written as $g = \tau\, c + b \cdot n(z) - q_z(b', a, z)\, b' + x$.

5.2 Sale Options

In addition to the non-contingent one-period bond available in the baseline model, we introduce options that give the government the right to sell its commodity at a given price in period $t+1$. The budget constraint can be written as $g = \tau\, c + w - q(b', a, z)\, b' - k(a, z)$, where total wealth w' in period $t+1$ is given by the sum of oil revenues x' and debt b'.[24] The introduction of sale options imply that oil revenues are given by $x' = \theta \cdot \max\{z', s(z)\}$, where θ is the constant quantity produced of the commodity. The derivative gives the government the option to sell at the maximum between spot price z' and a predetermined strike price $s(z)$. The strike price is set one period in advance, as the price for period $t+1$ that is expected at the time that the contract is signed $s(z) = \sum_{\{z'\}} \Gamma(z' \mid z)\, z'$. The cost $k(a, z)$ of the derivative is given by the expected dis-

[24] We can rewrite the model in terms of wealth to avoid introducing an additional state variable (see the Appendix). The same budget constraint is used in the case of forward selling, with a modification in how oil revenues x' are determined (discussed in the next subsection).

counted cash flow, valued with the stochastic discount factor previously described.[25] With partial hedging oil revenues are given by $x' = \theta\ (\xi \cdot z' + (1-\xi) \cdot \max\{z', s(z)\})$, where parameter ξ determines the extent to which options are used to cover oil revenues: ξ equal to one implies that derivatives are not used and ξ equal to zero implies full coverage (the cost of the option is adjusted accordingly).

5.3 Forward Sales

In addition to the non-contingent one-period bond available in the baseline model, we allow the government to set the price for its commodity one year in advance, following Borensztein et al. (2013). This works as follows: if the spot price of oil in period t is z, oil revenues in period $t+1$ will be given by $s(z) \cdot \theta$, where θ is the constant quantity produced of the commodity. The price $s(z)$ is set as the expected value for period $t+1$ with the information that is known at period t, written as $s(z) = \sum_{\{z'\}} \Gamma(z' \mid z) \cdot z'$, which is the expected value of z' in period t, when z is known.[26] The budget constraint is written in the same manner as in the case for sale options. As in the case of sale options, it is assumed initially that these contracts are available during default episodes.[27]

6 Quantitative Analysis

The objective is to provide a quantitative analysis of the consequences of using different financial derivatives that contribute to moderate the fluctuations of commodity revenues and their impact on the economy. We document how business cycle moments are modified with the introduction of new financial instruments. In particular, we document a reduction in the volatility of different macroeconomic variables, their correlation with the price of oil and the correlation of government expenditures with the business cycle. We then conduct an event analysis centered on episodes when there are large drops in commodity prices and compare the evolution of the baseline economy with an economy where the government uses different financial derivatives. In our welfare analysis we al-

[25]It was not possible to consider alternative valuation methods, such as a binomial model. We show below that for the baseline calibration, this instrument provides the lowest welfare gains in the baseline calibration, so we are not overestimating its relative advantage.

[26]It is straightforward to prove that, assuming an AR(1) process, the variance of $s(z)$ is lower than the variance of z.

[27]This will result, in our baseline calibration, in lower average debt levels sustained in equilibrium and more conservative *conditional* welfare gains as we discuss below, but no significant differences in terms of business cycle moments.

low for the possibility of partial indexation and hedging.[28]

6.1 Business Cycle Statistics

Tables 3-5 document the key business cycle statistics of the model under different scenarios. In addition to allowing the government to access different types of financial instruments we can recreate a scenario, starting from the baseline specification, where we eliminate the volatility of oil revenues. This exercise provides a benchmark in terms of the overall impact of oil-revenue volatility in the model economy (e.g., the amount of volatility that it generates in other macroeconomic variables, see Table 3).[29] It establishes the scenario where oil-revenue volatility is completely eliminated. For example, eliminating volatility in oil-revenues reduces the volatility in private consumption from 0.039 to 0.027.[30] The effect on government expenditures is more pronounced as its volatility decreases from 0.084 to 0.039 (Table 3).

Table 3. Business Cycle Moments: Standard Deviations.

standard deviation *log-detrended w/HP filter*	base model	no oil shocks	indexed bonds	forward sale	sale option
production output	0.030	0.024	0.028	0.028	0.029
consumption	0.039	0.027	0.035	0.035	0.036
govt. expenditures	0.084	0.039	0.070	0.066	0.074
labor	0.027	0.019	0.024	0.024	0.025
tax rate	0.021	0.008	0.020	0.020	0.020
trade balance/total output	0.010	0.009	0.022	0.021	0.014

The comparison with the alternative scenarios makes explicit the fact that although financial instruments can reduce the volatility of macroeconomic variables they cannot,

[28]In terms of business cycle moments and event analysis, partial indexation and hedging simply moderate the impact on the different macroeconomic variables relative to the full indexation or hedging case.

[29]For this exercise we adjust parameter θ so that the average of commodity revenues is the same as in the baseline specification.

[30]Schmitt-Grohé and Uribe (2015) estimate that terms of trade shocks account for approximately 12 percent of consumption volatility and 17 percent of output volatility in the case of Mexico (their Table 2). Pieschacon (2012), also for the case of Mexico, estimates that oil price shocks account for 21.3 percent of the variance of consumption at a 4-quarter horizon (her Table 1), while the shares are 12.5 and 16.8 percent, respectively, for tradable and non-tradable output. The proportions of volatility of consumption and production (non-oil) output explained by oil shocks in our model, approximately 29 and 22 percent, are somewhat higher but comparable to these empirical estimates.

as would be expected, completely eliminate the volatility induced by fluctuations in oil-revenues. Among the different financial instruments, we find that forward-selling is most effective in reducing volatility for different the macroeconomic variables. The use of sale options has the lowest effect in reducing volatility as their utilization reduces the impact of downward fluctuations, while they have no effect when spot prices are higher than the predetermined strike price of the option. The increased volatility in the trade balance, with the new financial instruments, reflects the ability to exploit access to international financial markets to smooth expenditures (this is further discussed below).

Table 4. Business Cycle Moments: Correlations.

correlation *log-detrended w/HP filter*	base model	no oil shocks	indexed bonds	forward sale	sale option
oil price and tax rate	-0.795	−−	-0.323	-0.453	-0.700
oil price and govt. exp.	0.859	−−	0.265	0.435	0.713
oil price and consumption	0.674	−−	0.233	0.437	0.578
oil price and prod. output	0.599	−−	0.192	0.368	0.507
govt. exp. and total output	0.932	0.914	0.707	0.800	0.892
govt. exp. and consumption	0.833	0.934	0.761	0.863	0.820
tax rate and prod. output	-0.799	-0.669	-0.700	-0.762	-0.783
tax rate and total output	-0.879	-0.669	-0.670	-0.742	-0.850
prod. output and int. rate	-0.234	-0.272	-0.206	-0.261	-0.235

The baseline model delivers a strong negative correlation between oil prices and the tax rate and strong positive correlation between oil prices and government expenditures. When the price of oil falls, the government increases tax rates to finance government expenditures, given that taxation is distortionary it will also find optimal to adjust government expenditures (Table 4).[31] The negative correlation between the tax rate and production output results in part from its distortionary effect on labor supply and from the higher tax rates during periods of low aggregate productivity (see the discussion of tax-rate policy functions in Section 3). Comparing the impact of different financial instruments,

[31]The procyclicality of tax rates and public expenditures is already present in sovereign default models without commodity-revenues (Cuadra et al., 2010; Hatchondo et al., 2012): adverse aggregate productivity shocks increase the likelihood of default, international investors are less willing to lend making government expenditures and private consumption positively correlated with output. A similar intuition applies to the role of commodity-revenues in our model. The correlation between tax changes and total output in our model is -0.46, close to the correlation estimated for Mexico by Vegh and Vuletin (2012). In their data, this correlation is driven by value added tax rates (see their Figs. 13 and 14). Using the tax-rate data from Anton-Sarabia (2005) for Mexico, for the period 1993-2001 for which different measures of both effective tax rates on consumption and labor income are available, the standard deviation of the sum (represented by the total tax rate in our model), is between 0.013 and 0.022, compared to 0.021 in our baseline model.

indexed bonds are most effective in reducing the correlation of different macroeconomic variables with the price of oil. Furthermore, indexed bonds are most effective in reducing the correlation of government expenditures with total output and private consumption.

Table 5. Business Cycle Moments: Averages.

average **(levels)**	base model	no oil shocks	indexed bonds	forward sale	sale option
government expenditures	0.159	0.159	0.160	0.159	0.159
private consumption	0.628	0.630	0.630	0.630	0.629
tax rate	0.156	0.155	0.154	0.155	0.156
debt/total output ratio	-0.247	-0.238	-0.164	-0.235	-0.238

In addition to the consequences in terms of the correlations and volatilities of the different variables in the model, the introduction of the alternative financial instruments has an effect on their average values in the stochastic steady state (Table 5). In particular, in some cases there is a slight increase in average consumption and government expenditures as well as a reduction in the average tax rate. These results are driven, in our baseline parameterization, by the fact that lower average debt levels are sustained in the new stochastic steady states, where sustainable levels of debt are determined by the incentives to default. The difference is most significant and natural for indexed bonds. The introduction of indexed debt reverses the incentives to default with respect to the price of oil: in the baseline specification the government has an incentive to default when the price of oil is low (as discussed in Section 3). With indexed debt the incentives to default become stronger with a higher price of oil, which is associated with larger debt payments.[32] Given these increased incentives to default, the government is able to sustain lower average levels of debt, this implies that lower average tax rates are needed and that more resources can be used for public expenditures (the lower tax rate is associated with higher private consumption).[33]

The introduction of derivatives implies a welfare gain both in autarky and with access to international financial markets.[34] The lower average debt levels sustained with derivatives in our baseline parameterization can be explained by the fact that they allow

[32]We can observe how the area of default becomes larger for high oil prices as we increment the level of debt indexation, eventually it becomes larger than the area of default with low oil prices.

[33]We discuss the welfare implications of this result below, contrasting the welfare results in the new stochastic steady state with those that consider the transition.

[34]We assume that contracts are available during periods of exclusion from debt markets. In an alternative version of the model with forward sales we assume the government jointly defaults on debt and this

to partially smooth the shocks in autarky (relative to the baseline situation where no instruments of any kind are available), compared to the situation with access to financial debt markets where the government already has an instrument available and the marginal gain generated by an additional instrument is relatively smaller.[35] In other words, autarky becomes a relatively less costly situation and incentives to default are then relatively stronger, reducing the average levels of debt the government is able to sustain (a similar intuition applies to the case with no oil shocks). Additionally, we find that the welfare gains associated with forwards during autarky, relative to the situation with access to financial markets, are higher than with options.[36] This implies that default is relatively less costly with forward sales than with sale options, therefore the average levels of debt that the government is able to sustain are lower (although this difference of -0.235 vs. -0.238 is rather small, Table 5).

6.2 Event Analysis: Large Drops in Oil Prices

Next we document how the economy reacts to commodity price fluctuations under different scenarios in terms of access to financial instruments. In particular, we simulate the alternative versions of our model and register the evolution of the main macroeconomic variables in front of drops in the price of oil that are larger than 30 percent when the price of oil is equal or above it unconditional mean.

The comparison of the baseline model and the model with forward-selling is shown in Figure 3. With an average fall in the price of oil of 50 percent, the government increases the tax rate by approximately 2 percentage points in the baseline scenario and reduces government expenditures by 14.7 percent.[37] The increase in the tax rate trans-

derivative (the government would never default on an option, it can simply select not to execute it), in this scenario there will be incentives to renege on debt and forward sales in situations where the hedging strategy results in a very negative payoff and debt is large. However, the temporary autarky situation is more costly if derivatives are not available. Results with the alternative assumption are available upon request, in terms of business cycle implications the differences are negligible.

[35] On the other hand, reducing uncertainty may work in the opposite direction: for example, eliminating fluctuations both in the price of oil and in production efficiency results in higher average debt levels (and no default since there are no shocks) of approximately 4 percentage points of total output. In our welfare analysis we consider an alternative parameterization with a higher discount parameter and find that this effect dominates, and the introduction of forward sales leads to (marginally) higher average levels of debt.

[36] We can verify this argument by introducing derivatives in the baseline model and compare welfare in autarky when μ (the probability of returning to financial markets) is zero. We find that welfare with forward sales is higher in permanent autarky than with sale options.

[37] Exploiting a VAR methodology for the case of Mexico, Pieschacon (2012) estimates that for a 20 percent quarterly increase in the price of oil, private consumption increases as much as 2 percent, while government purchases increase by almost 4 percent (Fig. 2 in Pieschacon, 2012). These results are slightly more moderate but comparable in magnitude with our baseline annual model, with average falls of 5.6 per-

lates directly into lower labor, production output and consumption. The use of forward-selling allows the government to smooth the adjustment in tax rates and government expenditures, resulting also in a smoother behavior for consumption, labor and production output. In the baseline scenario, the government slightly increases debt with a fall in oil prices (becomes more negative), while with forward-selling the government foresees a lower level of commodity revenues in the next period (the hedging strategy implies that the fall in oil revenues generated by the drop in oil prices is postponed one period), and therefore initially reduces the debt level.[38] The ability of the government to increase debt is determined by the level of debt at the time of the shock; if it occurs when the debt level is relatively low there is more margin to increase debt (not shown).

In Fig. 4 we compare the distribution of the percentage falls in consumption and government expenditures to contrast the likelihood of very negative events; the probability of large drops in both variables is higher in the baseline scenario. With access to sale-options the results are similar in terms of the ability of the government to smooth the evolution of the main macroeconomic variables (Figure 5). Additionally, upon the drop in oil prices, the government also chooses to slightly reduce debt. With indexed bonds however, the fall in oil-revenues has the same timing as in the baseline scenario (last panel, Fig. 6), and in both situations the government increases debt slightly. The increase in the volatility of the trade balance reflects the increased capacity for the government to smooth expenditures.[39]

6.3 Partial Bond Indexation

With commodity-indexed bonds the budget constraint of the government is written as $g = \tau\, c + b \cdot n(z) - q_z(b', a, z)\, b' + x$, where $n(z) = \nu + (1-\nu)\, z$. Parameter $\nu \in [0, 1]$ determines the extent to which debt is indexed to commodity prices: a proportion $(1-\nu)$ of the payment promised by the bond is indexed to the price of the commodity z. The price of these bonds is given by $q_z(b', a, z)$, as previously described.[40]

cent in consumption and 14.7 in government expenditures in front an average oil-price drop of 50 percent.

[38]Note the timing notation for the asset variable in our model. Given our calibration procedure, as previously discussed, there are not significant movements in interest rates. This is further discussed in the Appendix.

[39]For a discussion of a similar result with GDP-indexed bonds see Hatchondo and Martinez (2012).

[40]Here we illustrate the mechanisms behind debt indexation. In our welfare analysis we allow for the possibility of partial indexation as well as partial hedging with derivatives. Partial hedging generates more modest results (in a monotonic manner) in terms of reducing volatility in macroeconomic variables and their correlation with oil prices relative to full hedging (the intuition is straightforward).

We can rewrite the budget constraint as $g - \tau\, c + q_z(b', a, z)\, b' = b \cdot n(z) + x$, where the left hand side of the expression includes control variables in any given period, while the terms on the right hand side $b{\cdot}n(z){+}x$, which include oil revenues plus debt payments, are predetermined or depend on the exogenous price of oil z in every period. In Figure 7B we graph, as a function of the level of indexation, the standard deviation of the interest rate in the left panel, and the standard deviation of oil revenues plus debt payments in the right panel. We observe that indexation can generate a significant increase in the volatility of bond prices, inherited from the high volatility in commodity prices. Fluctuations in oil revenues, however, are offset by payments on indexed bonds generating a reduction in the volatility of the total sources of financing directly linked to the price of oil. Due to the fall in volatility of the total sources of financing linked to oil prices, there is a reduction in the volatility of private consumption and government expenditures, as well as in the correlation of the tax rate and government expenditures with oil prices (see Fig. 7A).

6.4 Welfare Analysis

In standard business cycle models featuring a representative household with risk averse preferences, the inability to insure against fluctuations in aggregate consumption implies a loss in welfare.[41] These welfare losses however are not quantitatively large in a representative agent framework. For the U.S., eliminating fluctuations in the cyclical component of aggregate consumption is equivalent to giving the representative household an increase of consumption of less than 0.1 percent across all dates and states of the world. Pallage and Robe (2003) estimate that removing consumption volatility in the least developed countries is equivalent to increasing consumption by approximately 0.3 percent in perpetuity.[42] As sometimes stressed in the literature, estimates of the welfare costs of economic fluctuations should not be considered in absolute terms. For example, welfare losses are larger when we consider idiosyncratic shocks and liquidity constraints faced by individuals over the business cycle rather than a representative household.

With these caveats in mind we conduct a welfare analysis for all instruments under different versions of the model and using two measures of welfare (defined and explained below).[43] In addition to the baseline model, we consider an alternative parameterization with a higher discount factor. We also consider a model without sovereign default, where

[41] This brief discussion partially builds on Pallage and Robe (2003).

[42] These results are based on the median welfare computations considering the observed volatility of consumption for a set of African economies and a risk aversion parameter of 2.5 (*basic model*, Table 2).

[43] We follow Schmitt-Grohé and Uribe (2007) in our definitions for analyzing welfare.

the debt limit is set exogenously.

6.4.1 Conditional Welfare Analysis

We first define welfare associated with the time-invariant stochastic allocation in the baseline model conditional on a particular initial state of the economy in period zero as:

$$\underline{v}_b = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \, u(c_t^b, g_t^b, 1 - l_t^b) \right]$$

where c_t^b, g_t^b and l_t^b denote the equilibrium contingent allocations for private consumption, government expenditures and labor in the baseline specification of our model, where only non-contingent debt is available.

In the same manner, the welfare conditional on an initial state of the economy in period zero, under an alternative scenario is given by (with its corresponding equilibrium contingent allocations):

$$\underline{v}_a = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \, u(c_t^a, g_t^a, 1 - l_t^a) \right]$$

At period zero we set the initial level of assets as the median of the stochastic steady state in the baseline model (this is our point of reference to evaluate the introduction of different financial instruments), while oil prices and aggregate productivity are at their mid-levels.[44]

Let γ denote the welfare gain of adopting a new financial instrument conditional on the initial state of the economy. This value is defined as the increment in the two consumption goods in all expected future states of the baseline economy such that the representative household is indifferent between the baseline economy and the alternative

[44] Here we depart from Schmitt-Grohé and Uribe (2007), who use the non-stochastic steady state as the initial state of the economy. In our model the non-stochastic steady state is not the most relevant point of reference, we want to evaluate the welfare gains for a government that faces uncertainty and decides to implement a hedging or indexation strategy, in this sense we are in line with Borensztein et al. (2013). Furthermore, the level of assets in the non-stochastic steady state could imply default in the stochastic model. Additionally we depart from Schmitt-Grohé and Uribe (2007) in that we have to consider two consumption goods.

economy (with access to additional financial instruments):

$$\underline{v}_a = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \, u((1+\gamma) \cdot c_t^b, (1+\gamma) \cdot g_t^b, 1 - l_t^b) \right]$$

Thus γ is defined as the value that implies this indifference.

6.4.2 Unconditional Welfare Analysis

Next we define Δ as the *unconditional* welfare gain of introducing an alternative or additional financial instrument, formally:

$$\mathbb{E}\left[\underline{v}_a\right] = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \, u((1+\Delta) \cdot c_t^b, (1+\Delta) \cdot g_t^b, 1 - l_t^b) \right]$$

This measure considers the expected welfare, where expectations are with respect to the ergodic distribution of the variables in the stochastic steady state. This measure does not take into account the transition towards the new stochastic steady state (this is done by the conditional welfare measure). This difference is relevant since, as we have shown previously, the introduction of alternative financial instruments can affect the average levels of debt that are sustained in equilibrium. In particular, if the government has to reduce its average levels of debt during the transition (initially this will be associated with higher average tax rates) this will imply a potential reduction of the welfare gains, whereas if the government is able to increase levels of debt then there is an additional potential gain in welfare.

6.4.3 Welfare Results

Table 6A reports, for the baseline model and calibration, the optimal level of hedging or indexation according to the *conditional* welfare criteria as well as the *unconditional* welfare gain associated with this particular level. In the case of forward sales, for example, full hedging is optimal while only a small amount of indexation is optimal.[45] From an *unconditional* welfare perspective, in all cases full hedging and indexation generate the largest gains in the new stochastic steady states. However, in the case of indexed bonds full indexation would imply that the government needs to considerably reduce average

[45] Partial indexation implies that the volatility of the different variables will not be reduced to the levels reported in Table 3. For example, the volatility of government expenditures with 10 percent indexation is 0.080 compared to 0.070 with full indexation.

levels of debt, as reported in Table 5, which actually generates a welfare loss if we consider the transition. Nevertheless, at the optimal level of debt indexation which is 1/10, the negative effects generated by the fact that the government has to reduce average debt levels are slightly outweighed by the gains from the reduction in volatility.[46] In the case of sale options, the costs associated with this derivative also contribute to reduce the welfare gains in the baseline parameterization.

Table 6A. Conditional and Unconditional Welfare Analysis.

baseline calibration *low discount factor* ($\beta = 0.85$)	indexed bonds	forward sale	sale option
optimal indexation/hedging	0.10	1.00	0.00
conditional welfare $(1+\gamma)$	1.0001	1.0007	—
unconditional welfare $(1+\Delta)$	1.0008	1.0035	—

We also consider an alternative calibration of the baseline model with a discount factor of 0.95, more in accordance with values in the standard business cycle literature (Table 6B).[47] Relative to the baseline calibration, from a *conditional* welfare perspective a higher discount factor gives more weight to the gains in the new stochastic steady state and the optimal levels of indexation and hedging are higher and the conditional welfare gains are larger.

Table 6B. Conditional and Unconditional Welfare Analysis.

alternative calibration *high discount factor* ($\beta = 0.95$)	indexed bonds	forward sale	sale option
optimal indexation/hedging	0.20	1.00	1.00
conditional welfare $(1+\gamma)$	1.0005	1.0029	1.0010
unconditional welfare $(1+\Delta)$	1.0009	1.0021	1.0013

[46] With this rate of indexation average debt levels are reduced by less than 1/10 of a percentage point relative to total output.

[47] To maintain the average debt levels of the baseline calibration we need to increase the efficiency cost of default parameter ω to 0.075, this is in the upper part of the range of the costs generally estimated in the sovereign default literature. Average debt levels as a ratio over total output are 1 percentage point higher with forward sales, this accounts for conditional welfare gains being bigger than unconditional welfare gains. Welfare gains will, of course, also depend on the relevance of commodity revenues and their volatility.

Finally, we report the results for a version of the model with an exogenous debt limit. We set the exogenous debt limit equal to the largest level of debt observed in the baseline model, then we set the efficiency cost of default parameter ω equal to a large number, so that default does not occur in equilibrium. For this version of the model we also use a higher discount factor of 0.95 as it is no longer a sovereign default model. The most significant difference relative to the results of the sovereign default model with high discount factor are for the indexed bonds (Table 6C). This instrument had the most important impact on the sustained average levels of debt in the model with sovereign default (Table 5), whereas in the model with an exogenous debt limit this mechanism no longer comes into play (this effect had a negative impact on welfare as it forced the government to reduce the amount of debt). Thus, in the version of the model with an exogenous debt limit full indexation becomes optimal, associated with larger welfare gains as indexed debt is the most effective in reducing the volatility of government expenditures, private consumption and labor.

Table 6C. Conditional and Unconditional Welfare Analysis.

alternative model *exogenous debt limit* ($\beta = 0.95$)	indexed bonds	forward sale	sale option
optimal indexation/hedging	1.00	1.00	0.90
conditional welfare $(1+\gamma)$	1.0036	1.0028	1.0010
unconditional welfare $(1+\Delta)$	1.0045	1.0018	1.0009

Note that our specification for an exogenous debt limit is different from the one analyzed by Borensztein et al. (2015). In the baseline version of their model insurance may allow the country to considerably increase the level of borrowing; the country can issue non-defaultable debt subject to a limit on the fraction of its output that it can pledge in repayment to foreign lenders. They find that introducing bonds that provide insurance against natural disasters allows the country to increase its external borrowing from approximately 30 percent of GDP to more than 60 percent of GDP (this results in a substantial conditional welfare gain).[48] The welfare gains in their model decrease with the discount factor, because most of the welfare gains come from relaxing the borrowing constraint and these gains are reduced when the consumer is more patient. In an alternative

[48]This result is similar in Borensztein et al. (2013); introducing the possibility of hedging against commodity price fluctuations allows an increment of debt from zero to 80 percent of income. This result generates the largest conditional welfare gains. However, they discuss how if the model is interpreted as one with overlapping generations and altruistic (but impatient) parents, introducing the possibility of hedging increases the welfare of current generations at the cost of future generations.

version of their model where default is possible the welfare gains are reduced to those generated by the insurance channel (which represent a fraction of a percentage point of annual consumption), since the new instrument has a relatively small impact on the default threshold (they provide a discussion of why it is not definitive a priori how the introduction of the new instrument affects the borrowing constraint for non-contingent debt).

7 Conclusion

We have analyzed a sovereign default model with endogenous fiscal policy to evaluate the macroeconomic consequences of using financial derivatives and commodity-indexed bonds to moderate the impact of fluctuations in commodity-related government revenues. We have documented how these instruments reduce the volatility of the different macroeconomic variables as well as their correlation with commodity prices.

Different instruments offer different trade-offs in terms of costs and benefits. An advantage of commodity linked bonds over futures contracts is that futures contracts may have relatively limited maturities available, whereas bonds can, in principle, be issued at longer-term maturities (Atta-Mensah, 2004; Daniel, 2001). The benefits of indexed debt, however, may be offset by significant fixed costs of setting up a market for a new debt product (Sandleris and Wright, 2013) as well as the possibility of being subject to low liquidity. Additionally, the use of derivatives may entail political costs if interpreted as speculative by the public. It has been suggested that international financial institutions may contribute to their use by promoting awareness and supporting risk management practices (Daniel, 2001; Caballero and Panageas, 2008).[49] However, investors and sovereign debtors may consider that existing markets for futures and options provide sufficient opportunities for insurance against commodity price fluctuations while financial innovation may encounter many potential obstacles (see Borensztein and Mauro, 2004).

[49] A possible concern is that investors may influence prices in financial markets. Fattouh et al. (2012) review the literature on the role of speculation in oil markets and find that the existing evidence is not supportive of an important role of financial speculation in driving the spot price of oil after 2003. Instead, they consider that there is strong evidence that the co-movement between spot and futures prices reflects common economic fundamentals rather than the financialization of oil futures markets. Knittel and Pindyck (2013) support the view that speculation had little, if any, effects on prices and volatility. Kilian and Murphy (2014) argue that the surge in oil prices during 2003-2008 was mainly driven by unexpected increases in world oil consumption, although speculative demand shifts may have played an important role during earlier price shock episodes in 1979, 1986 and 1990. Juvenal and Petrella (2015), on the other hand, argue that even though the recent oil price increase was mainly driven by global demand, the financialization process of commodity markets also played a role. We stress that the key assumption in our framework is that the behavior of commodity prices are taken as given by the small open economy.

There are several potentially interesting research possibilities for these issues. Hatchondo et al. (2012) exploit a sovereign default model to analyze the benefits of implementing a debt ceiling rule and demonstrate that lower debt levels allow the government to implement a less procyclical fiscal policy that reduces aggregate consumption volatility. Aguilar and Ramirez (2013), Kumhof and Laxton (2013), Medina and Soto (2013), Pieschacon (2012), Snudden (2013) evaluate the implications of different fiscal policy rules in models that incorporate the effect of commodity price fluctuations on public finances. Introducing the possibility of accumulating international reserves, or sovereign wealth funds, could provide quantitative guidance to the claim that the use of hedging instruments reduces the cost of opportunity implied by these assets and the possibility of evaluating other potential trade-offs.[50] Additional work is also necessary to study alternative transmission mechanisms from commodity price fluctuations to private sector production.[51] We leave these topics for future research.

[50] Van der Ploeg (2014) makes the case that a country managing natural resource wealth should establish three funds: an intergenerational sovereign wealth fund to smooth consumption across generations, a liquidity fund to deal with commodity price volatility, and an investment fund to control spending on domestic investment. Bianchi et al. (2014) extend a dynamic model of sovereign default with sudden-stop shocks in which the government faces the trade-off between the insurance benefits of reserves and the cost of keeping larger gross debt positions. Some of the extensions discussed would increase the computational burden in our model, given the additional endogenous state variable. We have abstracted from the impact of commodity prices on real exchange rates; Aizenman et al. (2012) find an important role for international reserves in stabilizing the real exchange rate in the presence of large commodity terms of trade shocks. Kohlscheen and O'Connell (2015) construct a default model with international reserves and credit; international reserves can provide an interim source of trade finance during periods of debt distress.

[51] Fernandez et al. (2015) and Shousha (2016) emphasize working capital requirements in the private sector as the financial channel through which fluctuations in interest rates are transmitted to the private sector. For the specification of this constraint see Neumeyer and Perri (2005).

8 References

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Fig. 1: Default Decision as a Function of Assets, Agg. Productivity and Oil Price

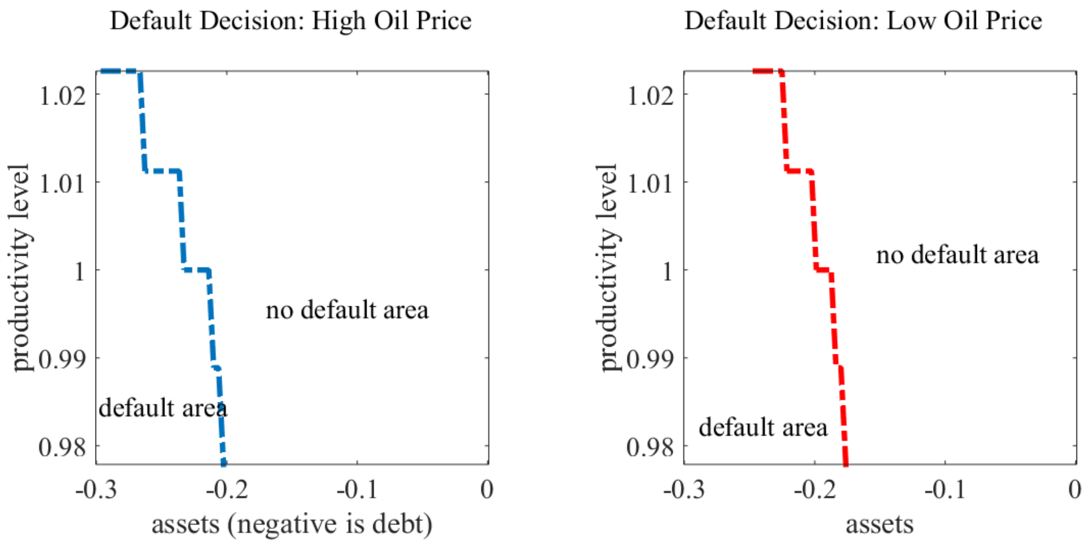


Fig. 2: Tax Rate as a Function of Assets, Agg. Productivity and Oil Price

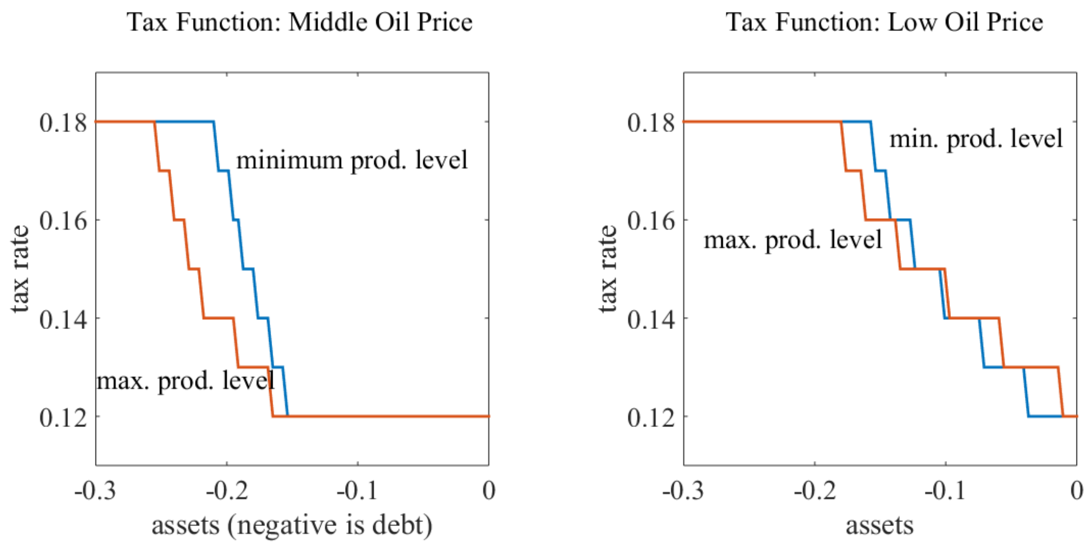


Fig. 3: Baseline and Forward Selling Models

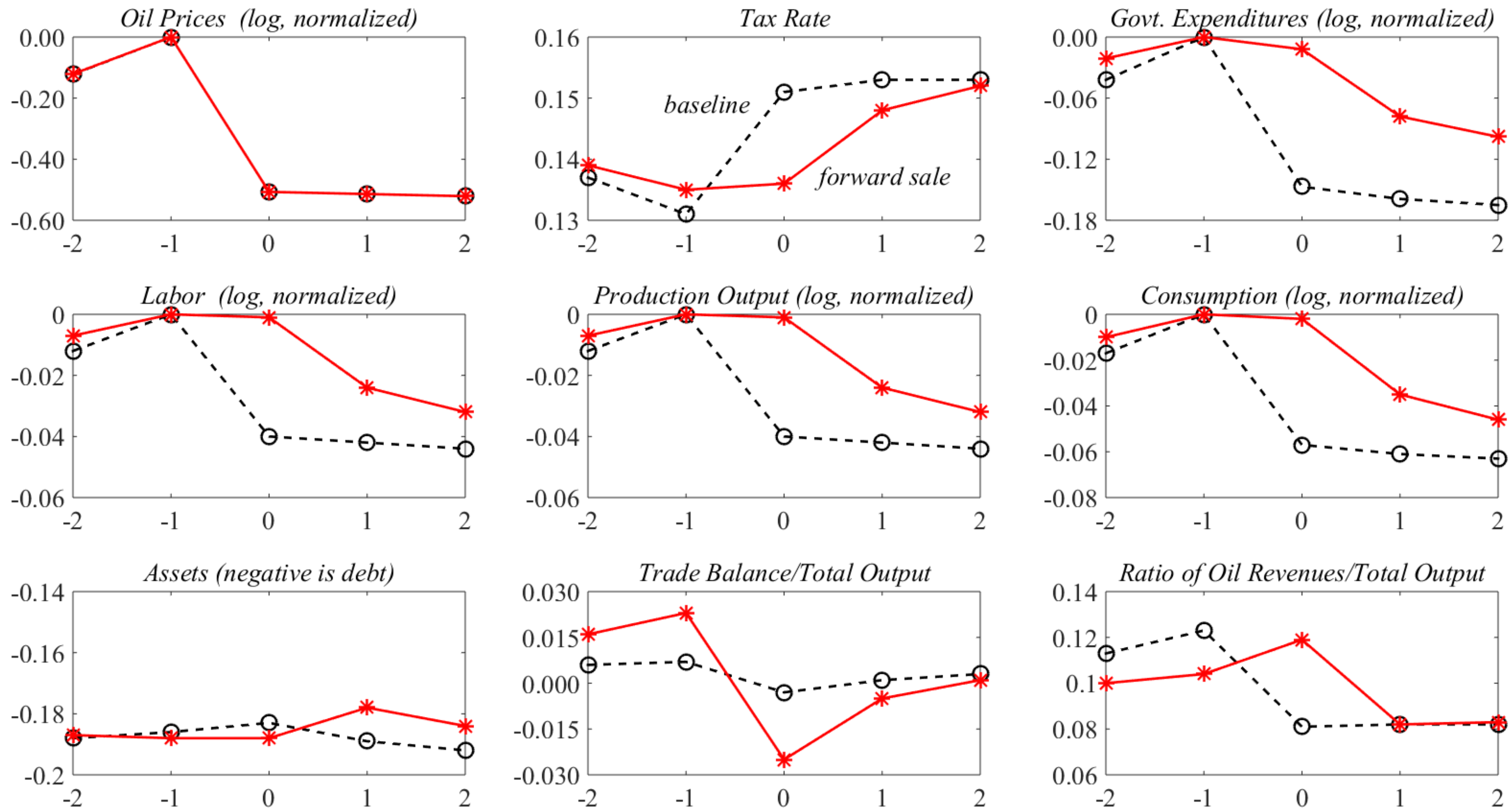


Fig. 4: Changes in Consumption and Govt. Expenditures Generated by the Fall in Oil Prices

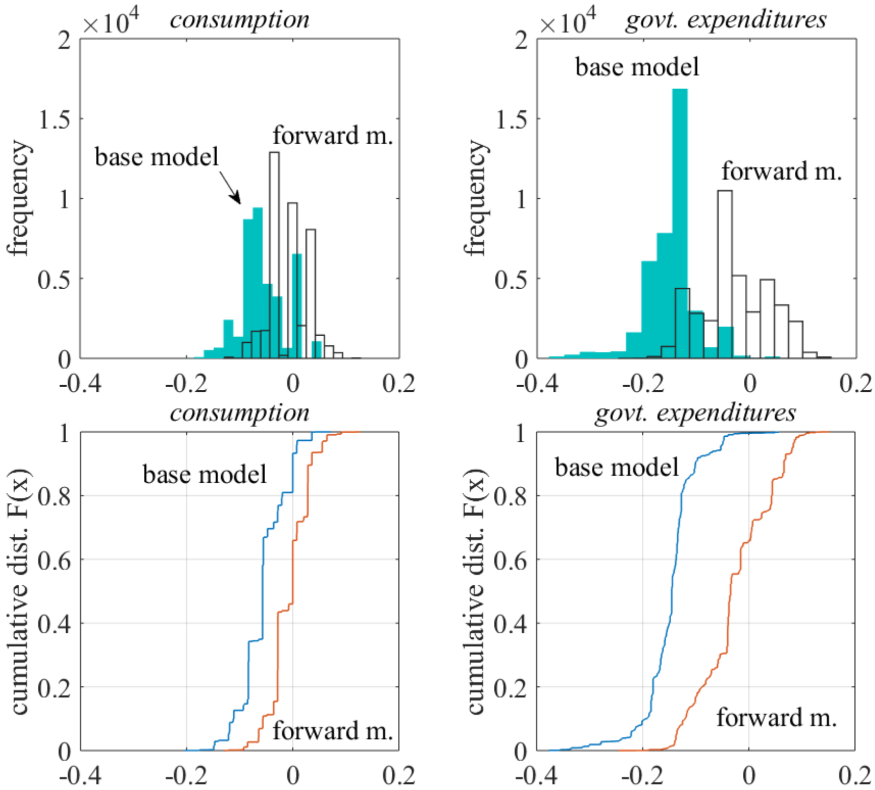


Fig. 5: Baseline and Option Selling Models

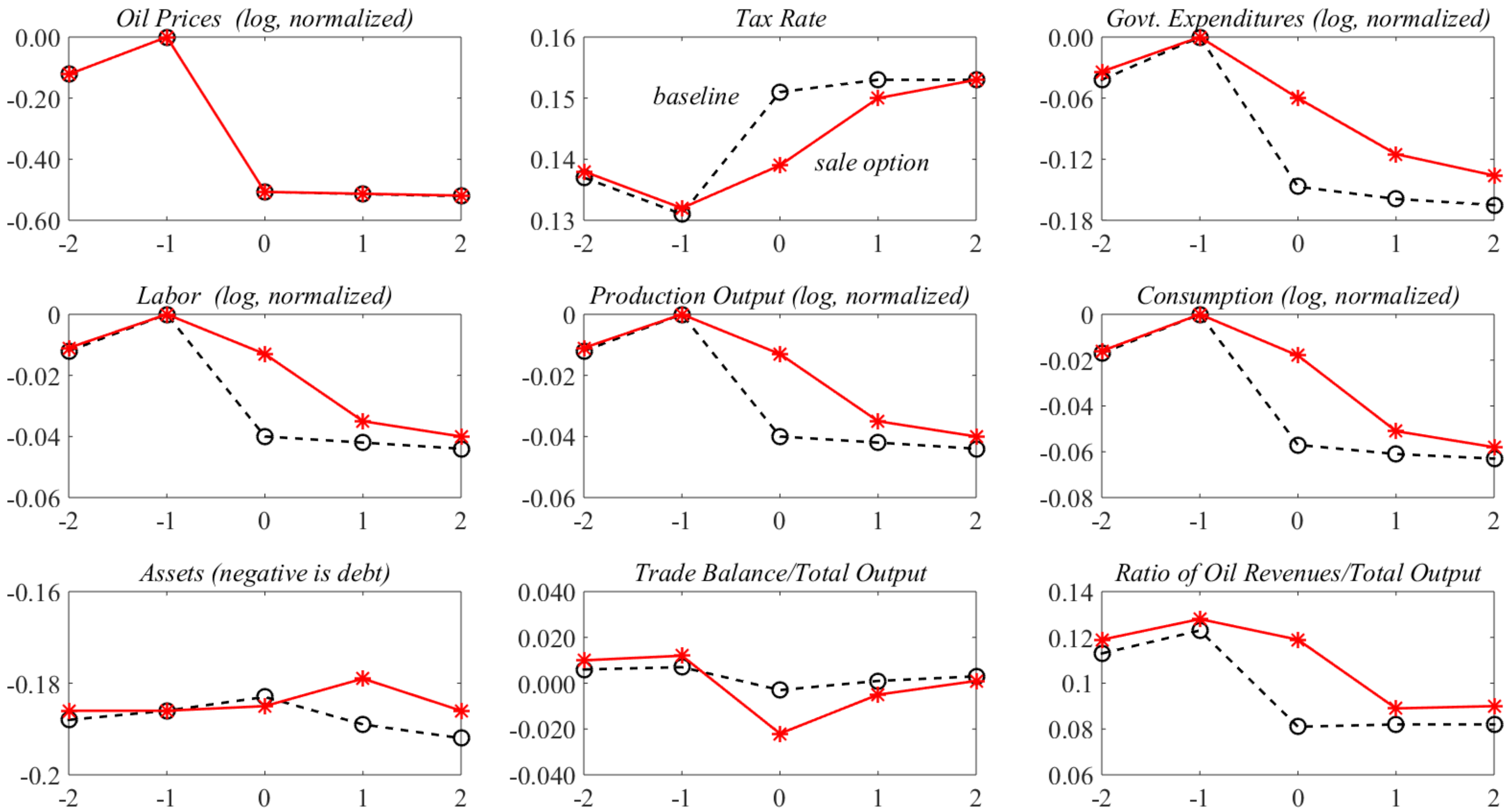


Fig. 6: Baseline and Indexed Bond Models

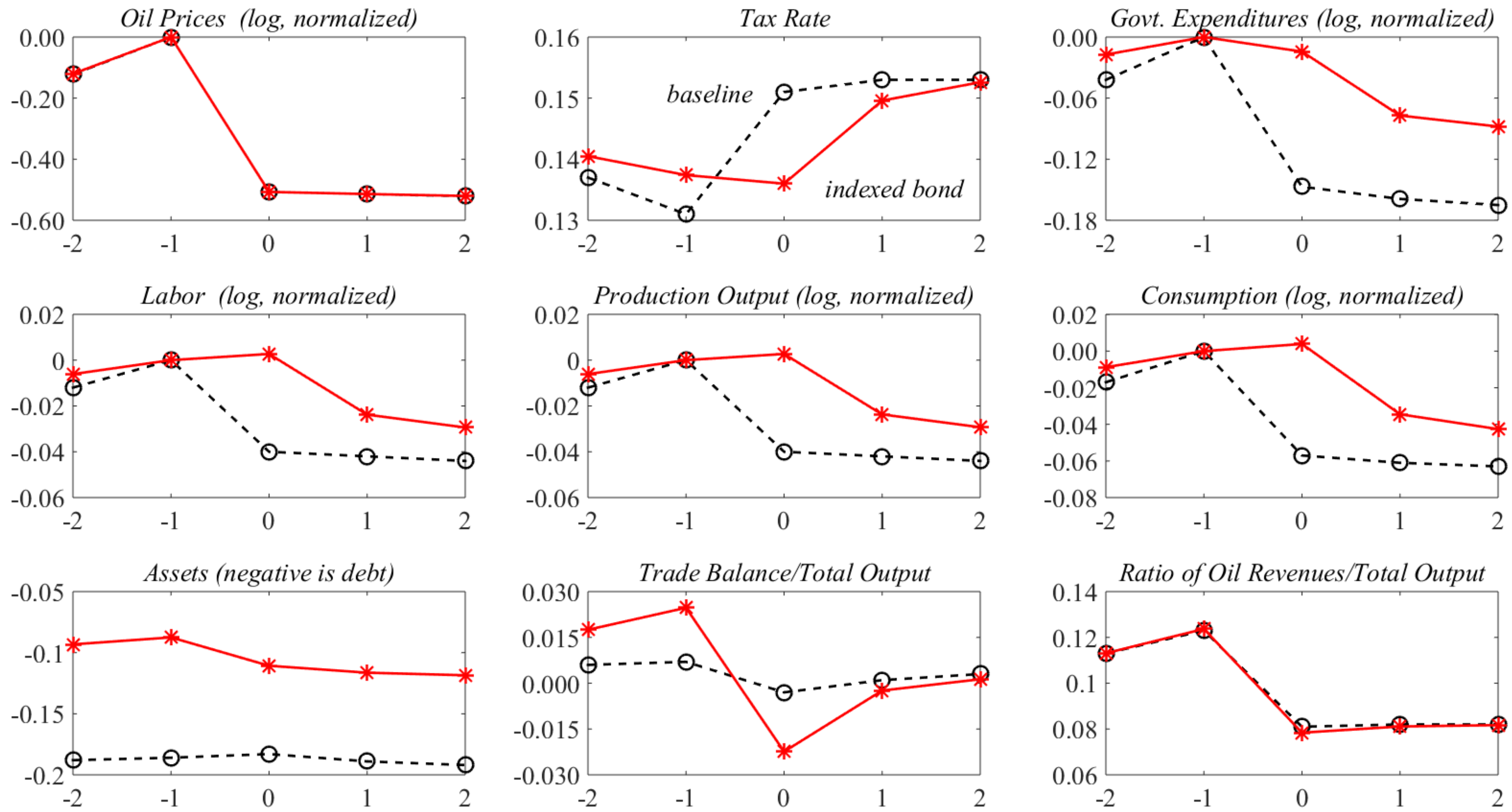


Fig. 7A: Properties of Indexed Bond Model

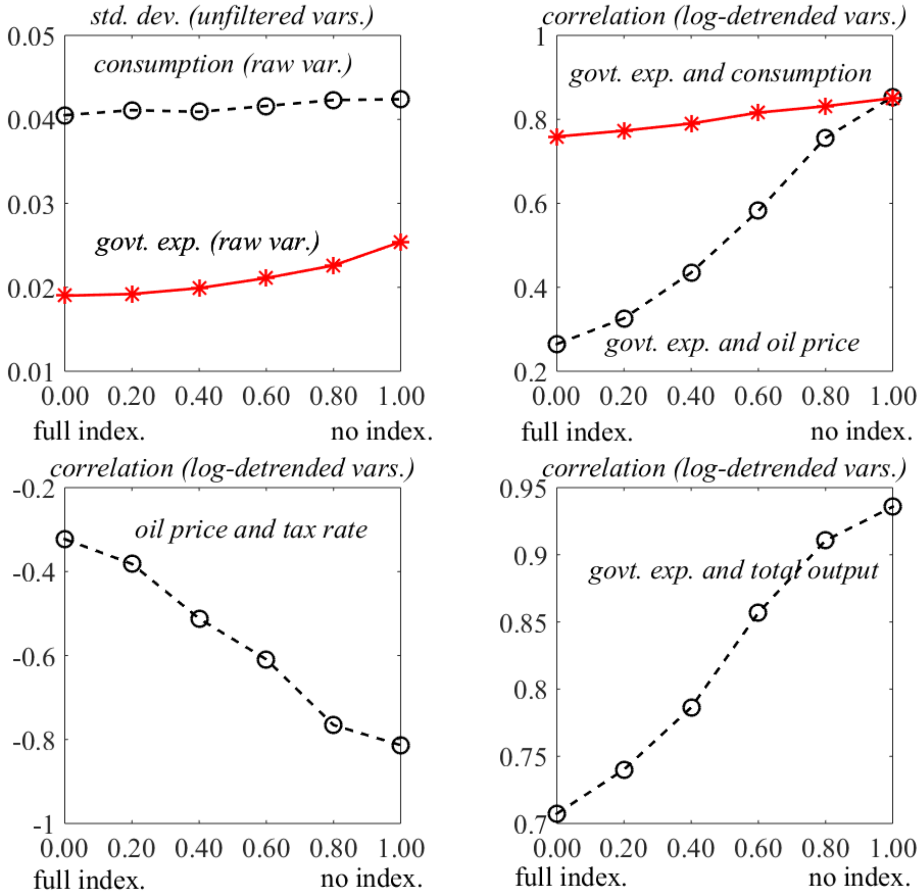
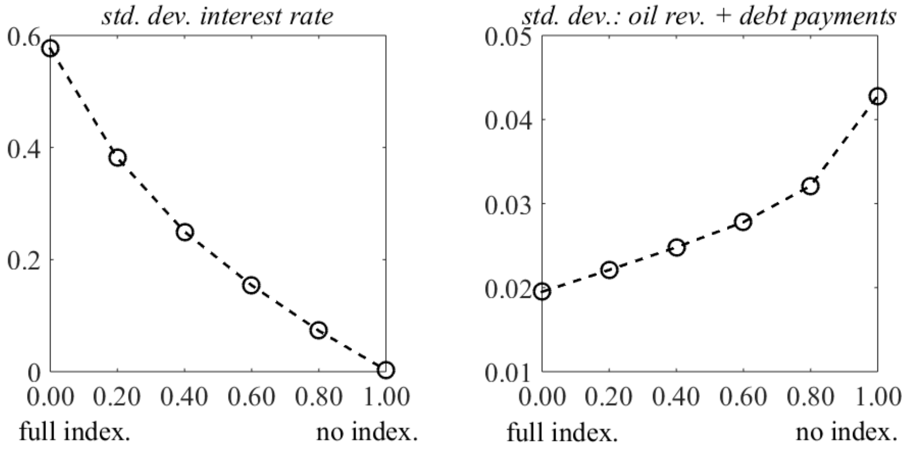


Fig. 7B: Properties of Indexed Bond Model



9 Appendix

9.1 Model in Terms of Wealth

We can rewrite the model in terms of a new state variable w, *total wealth*. This is useful since in the case of forward-selling, for example, we do not need to keep the contract price set in the previous period as an additional state variable. With access to financial markets, the dynamic problem can now be written as follows:

$$v_c(w, a, z) = \max_{\{g, w', \tau\}} u(c^*, g, 1 - l^*) + \beta \sum_{\{a', z'\}} \Lambda(a' \mid a) \, \Gamma(z' \mid z) \, \overline{v}(w', a', z')$$

subject to the optimal household functions $\{c^*(\,\cdot\,), l^*(\,\cdot\,)\}$. The government budget constraint is $g = \tau\, c + w - q(\,\cdot\,)\,(w' - x')$, where $w' = b' + x'$ is total wealth in the next period (assets, or debt, plus oil-revenues). With *forward-selling*, for example, oil revenues x' are known one period in advance and given by $x' = \theta \cdot \sum_{\{z'\}} \Gamma(z' \mid z)\, z'$.

When the government defaults its budget constraint becomes $g = \tau\, c + w$, where now we have $w' = x'$, the dynamic problem of the government is given by:

$$v_d(w, a, z) = \max_{\{g, \tau\}} u(c_d^*, g, 1 - l_d^*) + \beta \sum_{\{a', z'\}} \Lambda(\,\cdot\,)\, \Gamma(\,\cdot\,) \left\{ \mu\, \overline{v}(w', \,\cdot\,) + (1 - \mu)\, v_d(w', \,\cdot\,) \right\}$$

subject to the budget constraint under default and the optimal decisions of the household $\{c_d^*(\,\cdot\,), l_d^*(\,\cdot\,)\}$ when there are no access to international credit markets.

9.2 Numerical Solution Algorithm and Computation of Moments

The numerical solution algorithm is standard in the literature (see for example Aguiar and Gopinath, 2006; Arellano, 2008; Hatchondo et al., 2012). We describe it for the baseline model (grid sizes and the definition of state variables may change across model specifications).

- Assume an initial price function $q(\,\cdot\,)$ (a simple set of initial values is defined by the inverse of the risk free interest rate), value functions $v_c(b, a, z)$ and $v_d(a, z)$ and a default set (e.g. start with no default over the state space). Assets are defined on a grid with 240 points, aggregate productivity and commodity prices are defined on 5 point grids each, constructed following Tauchen (1986), and modified in the case of oil prices as described in the calibration section.

- The main decision variables for the government are the tax rate $\tau(b, a, z)$ and next-period assets $b'(b, a, z)$, both are functions of the state variables (b, a, z) (tax rates are defined on a grid with 7 points between 0.12 and 0.18).[52] For each point (b, a, z) and for every possible value of τ and b', we obtain $c(\,\cdot\,)$ and $l(\,\cdot\,)$ using the first order condition of the representative household and its budget constraint, then obtain $g(\,\cdot\,)$ from the government budget constraint, compute utility values and value functions. For every point (b, a, z) retrieve the optimal tax rate τ, policy function b' and the new default set $d(\,\cdot\,)$. Update value functions.

- Given the new default set recompute the bond price function.
- At this point policy function iteration is employed to accelerate convergence in the value function (the outer loop consists of value function iteration, the improvement in terms of convergence time is considerable as is typically expected).
- Return to the second step, and repeat until value function convergence is achieved (up to a determined tolerance level).

To compute the model generated moments, for each specification we simulate the model 1,000 times, with 500 periods for each simulation. The first 100 periods in each simulation are dropped to eliminate dependence of the results on the initial conditions (initial conditions can be arbitrary if this procedure is followed). Detrended variables (in logs) are computed employing an HP filter with a parameter value of 100. For welfare calculations we simulate the model 250 thousand times for each parameterization (conditional welfare values are verified with the value function solution).

9.3 Alternative Productivity Loss Specification During Default

In our baseline specification the aggregate productivity cost of default consists of a function $h(a)$ such that $h(a) = a - \omega$ when $a \leq \phi\,\bar{a}$, where ϕ is a parameter and $\bar{a}$ is the unconditional mean of aggregate productivity. When $a \geq \phi\,\bar{a}$, then $h(a) = \phi\,\bar{a} - \omega$. Relative to Arellano (2008), we have introduced a parameter ω, to match the ratio of debt to output in Mexico (see Table 2). This parameter shifts the level of productivity during default while maintaining the shape of the original function $h(a)$. We take the value of parameter ϕ from Cuadra et al. (2010) and set ω to match the ratio of public sector debt to total output for Mexico. In Fig. A1 we plot the resulting productivity during default under our baseline specification.

[52] Robustness exercises were conducted using a grid of 14 points for the tax rate, with no relevant differences in the results.

It is well established in the literature that sovereign default models face difficulty in jointly matching several moments related to sovereign debt and interest rates: average and volatility of sovereign interest rate spreads, frequency of default and average debt levels (see Aguiar and Gopinath, 2006; Arellano, 2008; Hatchondo and Martinez, 2009; Hatchondo et al., 2010; Yue, 2010; Roldan-Peña, 2012; Arellano and Ramanarayanan, 2012; Lizarazo, 2013; and the discussion in Mendoza and Yue, 2012). In the baseline calibration our target is the average debt level, given our primary interest in evaluating how the risk generated by commodity prices affects the possibility of the government to finance its expenditures. The volatility of interest rates, however, is limited at 0.003 while the average spread is 6 basis points. This volatility is limited even with the introduction of a stochastic discount factor based on Arellano and Ramanarayanan (2012). Given the annual calibration of our model, the volatility of the aggregate productivity process is considerably smaller than their value for a quarterly calibration with exogenous output, 0.005 compared to 0.017, respectively. The volatility of the stochastic discount factor, and its impact on sovereign interest rates, is determined by this parameter.[53]

Alternatively, we can consider a specification where productivity losses are convex-shaped (see Fig. A1). Under the alternative specification, aggregate productivity in default is given by the function $a_{def} = \max\{0.92 \cdot a_{cred}^{3.8}, 0.9239\}$, where the lower bound 0.9239 is set to match the lowest level of aggregate productivity under the baseline specification (see Figure A1), the value 3.8 generates the convex shape of productivity under default and increases interest rate volatility (different values are possible). Under this specification the efficiency cost of default is an increasing, convex function of productivity, thus introducing in a reduced form manner the mechanism studied by Mendoza and Yue (2012). The volatility of interest rates is 0.02 and average spreads are increased to 56 basis points. However, one notable drawback is that by making default relatively less costly when the level of productivity is high (compared to the baseline specification), default becomes relatively more attractive during good times, and the correlation of interest rates and production (non-commodity) output is negative but small at -0.023, compared to -0.234 in the baseline model, while the new average debt ratio is -0.164.

[53] Hatchondo et al. (2010) evaluate different algorithms to solve the models of Arellano (2008) and Aguiar and Gopinath (2006) and show the limited capacity for these models to generate volatility in interest rates with realistic mean levels of debt to output ratios. In particular, our results are in line with the low interest rate volatility in their solution of the model by Aguiar and Gopinath (2006) without trend shocks (see their Table 3). The model with shocks to the trend in aggregate productivity increases volatility of interest rates, although still at very low levels, but results in positive correlation of interest rates and output (see Table 3 in Hatchondo et al., 2010).

Fig. A1. Alternative Productivity Losses During Default